

# Dark Energy: Recent Developments\*

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## Abstract

A six parameter cosmological model, involving a vacuum energy density that is extremely tiny compared to fundamental particle physics scales, describes a large body of increasingly accurate astronomical data. In a first part of this brief review we summarize the current situation, emphasizing recent progress. An almost infinitesimal vacuum energy is only the simplest candidate for a cosmologically significant nearly homogeneous exotic energy density with negative pressure, generically called Dark Energy. If general relativity is assumed to be also valid on cosmological scales, the existence of such a dark energy component that dominates the recent universe is now almost inevitable. We shall discuss in a second part the alternative possibility that general relativity has to be modified on distances comparable to the Hubble scale. It will turn out that observational data are restricting theoretical speculations more and more. Moreover, some of the recent proposals have serious defects on a fundamental level (ghosts, acausalities, superluminal fluctuations).

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# 1 Introduction

On the basis of a rich body of astronomical observations there is now convincing evidence that the recent ( $z < 1$ ) Universe is dominated by an exotic nearly homogeneous dark energy density with *negative* pressure. The simplest candidate for this unknown so-called *Dark Energy* (DE) is a cosmological term in Einstein's field equations, a possibility that has been considered during all the history of relativistic cosmology. Independently of what this exotic energy density is, one thing is certain since a long time: The energy density belonging to the cosmological constant is not larger than the cosmological critical density, and thus *incredibly small by particle physics standards*. This is a profound mystery, since we expect that all sorts of *vacuum energies* contribute to the effective cosmological constant.

Since this is such an important issue for fundamental physics, astrophysics and cosmology, it should be of interest to indicate how convincing the evidence for this finding really is, or whether one should still remain sceptical. Much of this is based on the observed temperature fluctuations of the cosmic microwave background radiation (CMB), and large scale structure formation. When combined with other measurements a cosmological world model of the Friedmann-Lemaître variety has emerged that is spatially almost flat, with about 70% of its energy contained in the form of Dark Energy.

## 2 Luminosity-Redshift Relation of Type Ia Supernovae

The first serious evidence for a currently accelerating universe, and still the only direct one, came from the Hubble diagram for Type Ia supernovae, that are good – although not perfect – standard candles.

In an ideal Friedmann-Lemaître universe, it is easy to establish a relationship between the luminosity distance,  $D_L$ , of an ideal standard candle and the redshift,  $z$ , of the source. We recall that  $D_L$  is defined by  $D_L = (\mathcal{L}/4\pi\mathcal{F})^{1/2}$ , where  $\mathcal{L}$  is the intrinsic luminosity of the source and  $\mathcal{F}$  the observed energy flux. Astronomers use as logarithmic measures of  $\mathcal{L}$  and  $\mathcal{F}$  the *absolute and apparent magnitudes*<sup>1</sup>, denoted by  $M$  and  $m$ , respectively. The conventions are chosen such that the *distance modulus*  $m - M$  is related to  $D_L$  as follows

$$m - M = 5 \log \left( \frac{D_L}{1 \text{ Mpc}} \right) + 25. \quad (1)$$

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<sup>1</sup>Beside the (bolometric) magnitudes  $m, M$ , astronomers also use magnitudes  $m_B, m_V, \dots$  referring to certain wavelength bands  $B$  (blue),  $V$  (visual), and so on.

With the help of the Friedmann equations one can express the product of the Hubble parameter,  $H_0$ , and  $D_L$  as a function of  $z$  and the cosmological density parameters  $\Omega_X$  for the various species,  $X$ , of the energy-matter content, including Dark Energy. The comparison of the resulting theoretical *magnitude - redshift relation* with data leads to interesting restrictions for the cosmological  $\Omega$ -parameters. In practice often only  $\Omega_M$  and  $\Omega_\Lambda$ , the density corresponding to the cosmological constant  $\Lambda$ , are kept as independent parameters, where from now on the subscript  $M$  denotes non-relativistic (mostly cold dark) matter.

In view of the complex physics involved, it is not astonishing that type Ia supernovas are not perfect standard candles. Their peak absolute magnitudes have a dispersion of 0.3 - 0.5 mag, depending on the sample. Astronomers have, however, learned in recent years to reduce this dispersion by making use of empirical correlations between the absolute peak luminosity and light curve shapes. Examination of nearby SNe showed that the peak brightness is correlated with the time scale of their brightening and fading: slow decliners tend to be brighter than rapid ones. Using these and other correlations it became possible to reduce the remaining intrinsic dispersion, at least in the average, to  $\simeq 0.15\text{mag}$ . (For the various methods in use, and how they compare, see e.g. [1], [2], and references therein.) Other corrections, such as Galactic extinction, have been applied, resulting for each supernova in a corrected (rest-frame) magnitude.

After the classic papers [3], [4], [5] on the Hubble diagram for high-redshift Type Ia supernovae, published by the SCP and HZT teams, significant progress has been made (for reviews, see Refs. [6], [7]). The results, presented in Ref. [2], are based on additional data for  $z > 1$ , obtained in conjunction with the GOODS (Great Observatories Origins Deep Survey) Treasury program, conducted with the Advanced Camera for Surveys (ACS) aboard the Hubble Space Telescope (HST). In the meantime new results have been published. Perhaps the best high- $z$  SN Ia compilation to date are the results from the Supernova Legacy Survey (SNLS) of the first year [8]. The other main research group has also published new data at about the same time [9]. Fig. 1 shows the data points of Ref. [8] for the distance moduli relative to an empty uniformly expanding universe as a function of redshift. Also shown is the prediction of the best fit values of a six parameter  $\Lambda$ CDM model, using only the three-year WMAP data (see Sect. 4).

Possible systematic uncertainties due to astrophysical effects have been discussed extensively in the literature. The most serious ones are (i) *dimming* by intergalactic dust, and (ii) *evolution* of SNe Ia over cosmic time, due to changes in progenitor mass, metallicity, and C/O ratio.

To improve the observational situation a satellite mission called SNAP

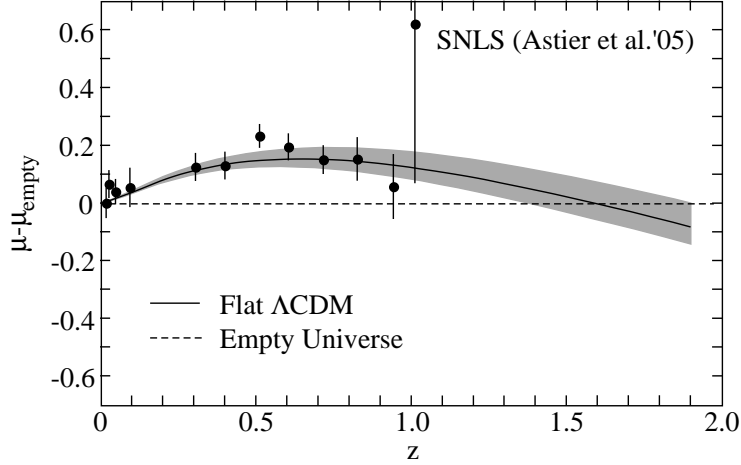


Figure 1: Distance moduli relative to an empty uniformly expanding universe (residual Hubble diagram) for SNe Ia of the SNLS data [8]. The shaded area shows the prediction for the luminosity-redshift relation from the  $\Lambda$ CDM model model fit to the three-year WMAP data only. (From Fig. 8 of Ref. [16].)

(“Supernovas Acceleration Probe”) has been proposed [10]. According to the plans this satellite would observe about 2000 SNe within a year and much more detailed studies could then be performed. For the time being some scepticism with regard to the results that have been obtained is still not out of place, but the situation is steadily improving.

Finally, we point out a more theoretical complication. In the analysis of the data the luminosity distance for an ideal Friedmann universe was always used. But the data were taken in the real inhomogeneous Universe. The magnitude-redshift relation in a perturbed Friedmann model has been derived in [24], and was later used to determine the angular power spectrum of the luminosity distance (the  $C_l$ ’s defined in analogy to (2)) [25]. One of the numerical results was that the uncertainties in determining cosmological parameters via the magnitude-redshift relation caused by fluctuations are small compared with the intrinsic dispersion in the absolute magnitude of Type Ia supernovae.

### 3 Microwave Background Anisotropies

Investigations of the cosmic microwave background have presumably contributed most to the remarkable progress in cosmology during recent years.

Beside its spectrum, which is Planckian to an incredible degree, we also can study the temperature fluctuations over the “cosmic photosphere” at a redshift  $z \approx 1100$ . Through these we get access to crucial cosmological information (primordial density spectrum, cosmological parameters, etc). A major reason for why this is possible relies on the fortunate circumstance that the fluctuations are tiny ( $\sim 10^{-5}$ ) at the time of recombination. This allows us to treat the deviations from homogeneity and isotropy for an extended period of time perturbatively, i.e., by linearizing the Einstein and matter equations about solutions of the idealized Friedmann-Lemaître models. Since the physics is effectively *linear*, we can accurately work out the *evolution* of the perturbations during the early phases of the Universe, given a set of cosmological parameters. Confronting this with observations, tells us a lot about the cosmological parameters as well as the initial conditions, and thus about the physics of the very early Universe. Through this window to the earliest phases of cosmic evolution we can, for instance, test general ideas and specific models of inflation.

### 3.1 Qualitative remarks

We begin with some qualitative remarks. Long before recombination (at temperatures  $T > 6000K$ , say) photons, electrons and baryons were so strongly coupled that these components may be treated together as a single fluid. In addition to this there is also a dark matter component. For all practical purposes the two interact only gravitationally. The investigation of such a two-component fluid for small deviations from an idealized Friedmann behavior is a well-studied application of cosmological perturbation theory (see, e.g., Ref. [12]).

At a later stage, when decoupling is approached, this approximate treatment breaks down because the mean free path of the photons becomes longer (and finally ‘infinite’ after recombination). While the electrons and baryons can still be treated as a single fluid, the photons and their coupling to the electrons have to be described by the general relativistic Boltzmann equation. The latter is, of course, again linearized about the idealized Friedmann solution. Together with the linearized fluid equations (for baryons and cold dark matter, say), and the linearized Einstein equations one arrives at a complete system of equations for the various perturbation amplitudes of the metric and matter variables. There exist widely used codes, e.g. CMBFAST [11], that provide the CMB anisotropies – for given initial conditions – to a precision of about 1%. A lot of qualitative and semi-quantitative insight into the relevant physics can, however, be gained by looking at various approximations of the basic dynamical system.

Let us first discuss the temperature fluctuations. What is observed is the temperature autocorrelation:

$$C(\vartheta) := \left\langle \frac{\Delta T(\mathbf{n})}{T} \cdot \frac{\Delta T(\mathbf{n}')}{T} \right\rangle = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\cos \vartheta), \quad (2)$$

where  $\vartheta$  is the angle between the two directions of observation  $\mathbf{n}, \mathbf{n}'$ , and the average is taken ideally over all sky. The *angular power spectrum* is by definition  $[l(l+1)/2\pi]C_l$  versus  $l$  ( $\vartheta \simeq \pi/l$ ).

A characteristic scale, which is reflected in the observed CMB anisotropies, is the sound horizon at last scattering, i.e., the distance over which a pressure wave can propagate until decoupling. This can be computed within the unperturbed model and subtends about half a degree on the sky for typical cosmological parameters. For scales larger than this sound horizon the fluctuations have been laid down in the very early Universe. These have been detected by the COBE satellite. The (gauge invariant brightness) temperature perturbation  $\Theta = \Delta T/T$  is dominated by the combination of the intrinsic temperature fluctuations and gravitational redshift or blueshift effects. For example, photons that have to climb out of potential wells for high-density regions are redshifted. One can show that these effects combine for adiabatic initial conditions to  $\frac{1}{3}\Psi$ , where  $\Psi$  is one of the two gravitational Bardeen potentials. The latter, in turn, is directly related to the density perturbations. For scale-free initial perturbations and almost vanishing spatial curvature the corresponding angular power spectrum of the temperature fluctuations turns out to be nearly flat (Sachs-Wolfe plateau).

On the other hand, inside the sound horizon before decoupling, acoustic, Doppler, gravitational redshift, and photon diffusion effects combine to the spectrum of small angle anisotropies shown in Figure 2. These result from gravitationally driven synchronized acoustic oscillations of the photon-baryon fluid, which are damped by photon diffusion.

A particular realization of  $\Theta(\mathbf{n})$ , such as the one accessible to us (all sky map from our location), cannot be predicted. Theoretically,  $\Theta$  is a random field,  $\Theta(\mathbf{x}, \eta, \mathbf{n})$ , depending on the conformal time  $\eta$ , the spatial coordinates  $\mathbf{x}$ , and the observing direction  $\mathbf{n}$ . Its correlation functions should be rotationally invariant in  $\mathbf{n}$ , and respect the symmetries of the background time slices. If we expand  $\Theta$  in terms of spherical harmonics,

$$\Theta(\mathbf{n}) = \sum_{lm} a_{lm} Y_{lm}(\mathbf{n}), \quad (3)$$

the random variables  $a_{lm}$  have to satisfy

$$\langle a_{lm} \rangle = 0, \quad \langle a_{lm}^* a_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l(\eta), \quad (4)$$

where the  $C_l(\eta)$  depend only on  $\eta$ . Hence the correlation function at the present time  $\eta_0$  is given by (2), where  $C_l = C_l(\eta_0)$ , and the bracket now denotes the statistical average. Thus,

$$C_l = \frac{1}{2l+1} \left\langle \sum_{m=-l}^l a_{lm}^* a_{lm} \right\rangle. \quad (5)$$

The standard deviations  $\sigma(C_l)$  measure a fundamental uncertainty in the knowledge we can get about the  $C_l$ 's. These are called *cosmic variances*, and are most pronounced for low  $l$ . In simple inflationary models the  $a_{lm}$  are Gaussian distributed, hence

$$\frac{\sigma(C_l)}{C_l} = \sqrt{\frac{2}{2l+1}}. \quad (6)$$

Therefore, the limitation imposed on us (only one sky in one universe) is small for large  $l$ .

A polarization map of the CMB radiation provides important additional information to that obtainable from the temperature anisotropies. For example, we can get constraints about the epoch of reionization. Most importantly, future polarization observations may reveal a stochastic background of gravity waves, generated in the very early Universe. The polarization tensor of an all sky map of the CMB radiation can be parametrized in temperature fluctuation units, relative to the orthonormal basis  $\{d\vartheta, \sin \vartheta d\varphi\}$  of the two sphere, in terms of the Pauli matrices as  $\Theta \cdot 1 + Q\sigma_3 + U\sigma_1 + V\sigma_2$ . The Stokes parameter  $V$  vanishes (no circular polarization). Therefore, the polarization properties can be described by a symmetric trace-free tensor on  $S^2$ . As for gravity waves, the components  $Q$  and  $U$  transform under a rotation of the 2-bein by an angle  $\alpha$  as  $Q \pm iU \rightarrow e^{\pm 2i\alpha}(Q \pm iU)$ , and are thus of spin-weight 2. “Electric” and “magnetic” multipole moments are defined by the decomposition

$$Q + iU = \sqrt{2} \sum_{l=2}^{\infty} \sum_m [a_{(lm)}^E + ia_{(lm)}^B] {}_2Y_l^m, \quad (7)$$

where  ${}_sY_l^m$  are the spin- $s$  harmonics.

As in Eq. (3) the multipole moments  $a_{(lm)}^E$  and  $a_{(lm)}^B$  are random variables and determine, similar to (2) and (5), the various angular correlation functions.

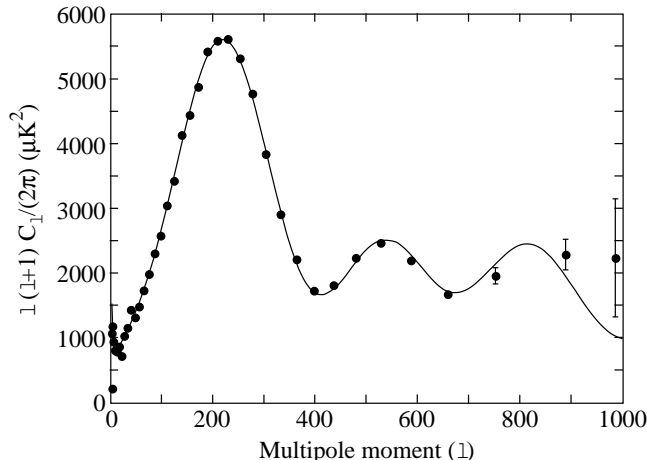


Figure 2: Three-year WMAP data for the temperature-temperature (TT) power spectrum. The black line is the best fit  $\Lambda$ CDM model for the three-year WMAP data. (Adapted from Figure 2 of Ref. [16].)

## 4 Observational Results and Cosmological Parameters

In recent years several experiments gave clear evidence for multiple peaks in the angular temperature power spectrum at positions expected on the basis of the simplest inflationary models and big bang nucleosynthesis[13]. These results have been confirmed and substantially improved by the first year WMAP data [14], [15]. Fortunately, the improved data after three years of integration are now available [16]. Below we give a brief summary of some of the most important results.

Fig. 2 shows the 3-year data of WMAP for the TT angular power spectrum, and the best fit (power law)  $\Lambda$ CDM model. The latter is a spatially flat model and involves the following six parameters:  $\Omega_b h^2$ ,  $\Omega_M h^2$ ,  $H_0$ , amplitude of fluctuations,  $\sigma_8$ , optical depth,  $\tau$ , and the spectral index,  $n_s$ , of the primordial scalar power spectrum. Fig. 3 shows in addition the TE polarization data [17]. There are now also EE data that lead to a further reduction of the allowed parameter space. The first column in Table 1 shows the best fit values of the six parameters, using only the WMAP data.

Combining the WMAP results with other astronomical data reduces the uncertainties for some of the six parameters. This is illustrated in the second column which shows the 68% confidence ranges of a joint likelihood analysis when the power spectrum from the completed 2dFGRS [19] is added. In



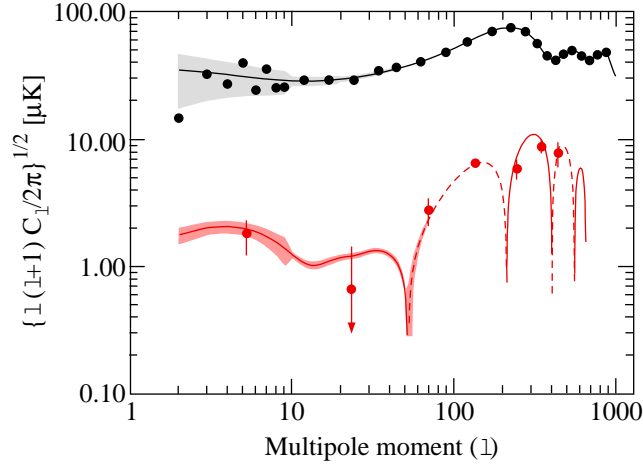


Figure 3: WMAP data for the temperature-polarization TE power spectrum. The best fit  $\Lambda$ CDM model is also shown. (Adapted from Figure 25 of Ref. [17].)

Ref. [16] other joint constraints are listed (see their Tables 5, 6). In Fig. 4 we reproduce one of many plots in [16] that shows the joint marginalized contours in the  $(\Omega_M, h)$ -plane.

The parameter space of the cosmological model can be extended in various ways. Because of intrinsic degeneracies, the CMB data alone no more determine unambiguously the cosmological model parameters. We illustrate this for non-flat models. For these the WMAP data (in particular the position of the first acoustic peak) restricts the curvature parameter  $\Omega_K$  to a narrow region around the degeneracy line  $\Omega_K = -0.3040 + 0.4067 \Omega_\Lambda$ . This does not exclude models with  $\Omega_\Lambda = 0$ . However, when for instance the Hubble constant is restricted to an acceptable range, the universe must be nearly flat. For example, the restriction  $h = 0.72 \pm 0.08$  implies that  $\Omega_K = -0.003^{+0.013}_{-0.017}$  and  $\Omega_\Lambda = 0.758^{+0.035}_{-0.058}$ . Other strong limits are given in Table 11 of Ref. [16], assuming that the equation of state parameter,  $w$ , has the value  $-1$  of vacuum energy. But even when this is relaxed, the combined data constrain  $\Omega_K$  and  $w$  significantly (see Figure 17 of [16]). The marginalized best fit values are  $w = -1.062^{+0.128}_{-0.079}$ ,  $\Omega_K = -0.024^{+0.016}_{-0.013}$  at the 68% confidence level.

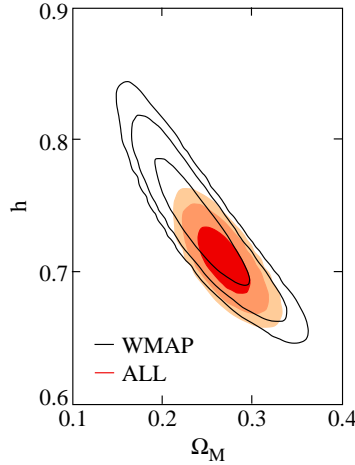


Figure 4: Joint marginalized contours (68% and 95% confidence levels) in the  $(\Omega_M, h)$ -plane for WMAP only (solid lines) and additional data (filled red) for the power-law  $\Lambda$ CDM model. (From Fig. 10 in [16].)

**Table 1.**

Parameter	WMAP alone	WMAP + 2dFGRS
$100\Omega_b h^2$	$2.233^{+0.072}_{-0.091}$	$2.223^{+0.066}_{-0.083}$
$\Omega_M h^2$	$0.1268^{+0.0072}_{-0.0095}$	$0.1262^{+0.0045}_{-0.0062}$
$h$	$0.734^{+0.028}_{-0.038}$	$0.732^{+0.018}_{-0.025}$
$\Omega_M$	$0.238^{+0.030}_{-0.041}$	$0.236^{+0.016}_{-0.029}$
$\sigma_8$	$0.744^{+0.050}_{-0.060}$	$0.737^{+0.033}_{-0.045}$
$\tau$	$0.088^{+0.028}_{-0.034}$	$0.083^{+0.027}_{-0.031}$
$n_s$	$0.951^{+0.015}_{-0.019}$	$0.948^{+0.014}_{-0.018}$

The restrictions on  $w$  – assumed to have no  $z$ -dependence – for a flat model are illustrated in Fig. 5 .

Another interesting result is that reionization of the Universe has set in at a redshift of  $z_r = 10.9^{+2.7}_{-2.3}$ . Later (Sect. 6.1) we shall add some remarks on what has been learnt about the primordial power spectrum.

It is most remarkable that a six parameter cosmological model is able to fit such a rich body of astronomical observations. There seems to be little room for significant modifications of the successful  $\Lambda$ CDM model. An exciting result is that the WMAP data match the basic inflationary predictions, and are even well fit by the simplest model  $V \propto \varphi^2$  (see Sect. 6 of [16]).

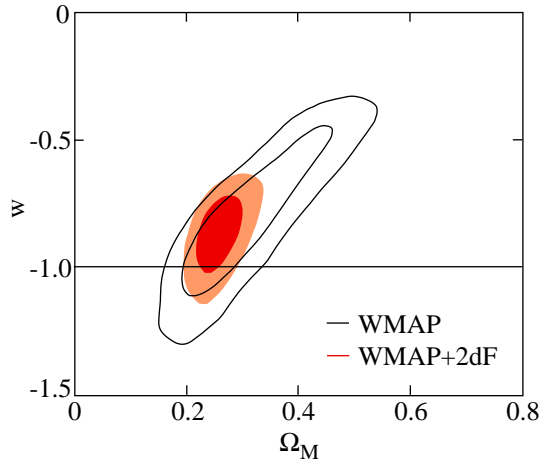


Figure 5: Constraints on the equation of state parameter  $w$  in a flat universe model when WMAP data are combined with the 2dFGRS data. (From Fig. 15 in [16].)

## 5 Dynamical Models of Dark Energy

If the vacuum energy constitutes the missing two thirds of the average energy density of the *present* Universe, we would be confronted with the following *cosmic coincidence* problem: Since the vacuum energy density is constant in time – at least after the QCD phase transition –, while the matter energy density decreases as the Universe expands, it would be more than surprising if the two are comparable just at about the present time, while their ratio was tiny in the early Universe and would become very large in the distant future. The goal of dynamical models of Dark Energy is to avoid such an extreme fine-tuning. The ratio  $w := p/\rho$  of this component then becomes a function of redshift.

In a large class of dynamic dark energy models the exotic missing energy with negative pressure is described by a scalar field, whose potential is chosen such that the energy density of the homogeneous scalar field adjusts itself to be comparable to the matter density today for quite generic initial conditions, and is dominated by the potential energy. This ensures that the pressure becomes sufficiently negative. It is not simple to implement this general idea such that the model is phenomenologically viable.

For an extensive recent review that contains a description of a variety of scalar field models, see Ref. [18]. It has to be emphasized that on the basis of the vacuum energy problem we would expect a huge additive constant for the

quintessence potential that would destroy the hole picture. Thus, assuming for instance that the potential approaches zero as the scalar field goes to infinity, has (so far) no basis. Apart of this and other fine tuning problems, I doubt that this kind of phenomenological models – with no natural field theoretical justification – will lead to an understanding of Dark Energy at a deeper level.

## 6 Alternatives to Dark Energy

In the previous sections we have discussed some of the wide range of astronomical data that support the following ‘concordance model’: The Universe is spatially flat and dominated by a Dark Energy component and weakly interacting cold dark matter. Furthermore, the primordial fluctuations are adiabatic, nearly scale invariant and Gaussian, as predicted in simple inflationary models. It is very likely that the present concordance model will survive phenomenologically.

A dominant Dark Energy component with density parameter  $\simeq 0.7$  is so surprising that it should be examined whether this conclusion is really unavoidable. In what follows I shall briefly discuss some alternatives that have been proposed.

### 6.1 Changes in the initial conditions

Since we do not have a tested theory predicting the spectrum of primordial fluctuations, it appears reasonable to consider a wider range of possibilities than simple power laws. An instructive attempt in this direction was made some time ago [20], by constructing an Einstein-de Sitter model with  $\Omega_\Lambda = 0$ , fitting the CMB data as well as the power spectrum of 2dFGRS. In the meantime, significant improvements in astronomical data sets have been made. In particular, the analysis of the three year WMAP data showed that there are no significant features in the primordial curvature fluctuation spectrum (see Sect. 5 of Ref. [16]). With the larger samples of high redshift supernovae and more precise information on large scale galaxy clustering, such models with vanishing Dark Energy are no more possible [21].

### 6.2 Inhomogeneous models

#### 6.2.1 Back reaction

It has recently been suggested [22], [23] that large scale perturbations may cause a large backreaction that could mimic dark energy and induce accel-

eration. The authors stressed that for investigating the effective dynamics averaging over a volume of size comparable with the present-day Hubble volume is essential. To decide on the basis of detailed calculations whether this is indeed possible is a very difficult task. However, from what we know about the CMB radiation it appears unlikely that there are such sizable perturbations out to very large scales.

The work by Kolb et al. [22], [23] triggered a lot of activity. We add some remarks about the ongoing discussion.

### 6.2.2 Exact inhomogeneous model studies

Effects of inhomogeneous matter distribution on light propagation were recently studied in the Lemaître-Tolman (LT) model, in order to see whether these can mimic an accelerated expansion.

The LT model is a family of spherically symmetric dust solutions of Einstein's equations. For these the magnitude-redshift relation can be worked out exactly.

As an example we mention Ref. [26], where it was shown that for  $\Lambda = 0$  the observed behavior of supernovae brightness can not be fitted, unless our position in the model universe is very special. In that case one has to analyze also other data, in particular the CMB angular power spectrum. At the time of writing, this has not yet been done, but is certainly underway.

## 6.3 Modifications of gravity

Since no satisfactory explanation of Dark Energy has emerged so far, possible modifications of GR, that would change the late expansion rate of the universe, have recently come into the focus of attention. The cosmic speed-up might, for instance, be explained by sub-dominant terms (like  $1/R$ ) that become essential at small curvature. Modified gravity models have to be devised such that to pass the stringent Solar System tests, and are compatible with the observational data that support the concordance model.

### 6.3.1 Generalizations of the Einstein-Hilbert action

The simplest generalization consists in replacing the Ricci scalar,  $R$ , in the Einstein-Hilbert action by a function  $f(R)$ . Note that this gives rise to fourth-order field equations. Applying a suitable conformal transformation of the metric, the action becomes equivalent to a scalar-tensor theory. In detail, if we define a new metric  $\tilde{g}_{\mu\nu} = \exp \left[ \sqrt{\frac{2}{3}} \kappa \varphi \right] g_{\mu\nu}$ ,  $\kappa^2 = 8\pi G$ , then the

action becomes

$$S = \int \left[ \frac{1}{2\kappa^2} R[\tilde{g}] - \frac{1}{2} \tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - V(\varphi) + L_{matter} \right] \sqrt{-\tilde{g}} d^4x, \quad (8)$$

where the potential  $V$  is determined by the function  $f$ . With this formulation one can, for instance, show that an arbitrary evolution of the scale factor  $a(t)$  can be obtained with an appropriate choice of  $f(R)$ . It is also useful to check whether a particular model passes Solar System tests (acceptable Brans-Dicke parameter). One should, however, bear in mind that the two mathematically equivalent descriptions lead to physically different properties, for instance with regard to stability. These issues and the application for specific functions  $f$  to Friedmann spacetimes, have recently been reviewed in [27].

We regard such modifications as quite ad hoc. Moreover, it has not yet been demonstrated that there are examples which satisfy all the constraints stressed above. The same can be said on generalizations [28], that include other curvature invariants, such as  $R_{\mu\nu} R^{\mu\nu}$ ,  $R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$ . In addition, such models are in most cases *unstable*, like mechanical Lagrangian systems with higher derivatives [29]<sup>2</sup>. An exception seem to be Lagrangians which are functions of  $R$  and the Gauss-Bonnet invariant  $G \equiv R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$ . By introducing two scalar fields such models can be written as an Einstein-Hilbert term plus a particular extra piece, containing a linear coupling to  $G$ . Because the Gauss-Bonnet invariant is a total divergence the corresponding field equations are of second order. This does, however, not guarantee that the theory is ghost-free. In Ref. [30] this question was studied for a class of models [28] for which there exist accelerating late-time power-law attractors and which satisfy the Solar System constraints. It turned out that in a Friedman background there are no ghosts, but there is instead *superluminal propagation* for a wide range of parameter space. This acausality is reminiscent of the Velo-Zwanziger phenomenon [31] for higher ( $> 1$ ) spin fields coupled to external fields. It may very well be that it can only be avoided if very special conditions are satisfied. This issue deserves further investigations.

### 6.3.2 First-order modifications of GR

The disadvantage of complicated fourth order equations can be avoided by using the *Palatini variational principle*, in which the metric and the sym-

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<sup>2</sup>This paper contains a discussion of a generic instability of Lagrangian systems in mechanics with higher derivatives, that was discovered by M. Ostrogradski in 1850.

metric affine connection (the Christoffel symbols  $\Gamma^\alpha_{\mu\nu}$ ) are considered to be independent fields.

It has long ago (1919) been shown by Palatini that for GR the Palatini formulation is equivalent to the Einstein-Hilbert variational principle, because the variational equation with respect to  $\Gamma^\alpha_{\mu\nu}$  implies that the affine connection has to be the Levi-Civita connection. Things are no more that simple for  $f(R)$  models:

$$S = \int \left[ \frac{1}{2\kappa} f(R) + L_{matter} \right] \sqrt{-g} d^4x, \quad (9)$$

where  $R[g, \Gamma] = g^{\alpha\beta} R_{\alpha\beta}[\Gamma]$ ,  $R_{\alpha\beta}[\Gamma]$  being the Ricci tensor of the independent torsionless connection  $\Gamma$ . The equations of motion are in obvious notation

$$f'(R) R_{(\mu\nu)}[\Gamma] - \frac{1}{2} f(R) g_{\mu\nu} = \kappa T_{\mu\nu}, \quad (10)$$

$$\nabla_\alpha^\Gamma (\sqrt{-g} f'(R) g^{\mu\nu}) = 0. \quad (11)$$

For the second of these equations one has to assume that  $L_{matter}$  is functionally independent of  $\Gamma$ . (It may, however, contain metric covariant derivatives.)

Eq. (11) implies that

$$\nabla_\alpha^\Gamma [\sqrt{-g} \hat{g}^{\mu\nu}] = 0 \quad (12)$$

for the conformally equivalent metric  $\hat{g}_{\mu\nu} = f'(R) g_{\mu\nu}$ . Hence, the  $\Gamma^\alpha_{\mu\nu}$  are equal to the Christoffel symbols for the metric  $\hat{g}_{\mu\nu}$ .

The trace of (10) gives

$$R f'(R) - 2f(R) = \kappa^2 T.$$

Thanks to this algebraic equation we may regard  $R$  as a function of  $T$ . In the matter-free case it is identically satisfied if  $f(R)$  is proportional to  $R^2$ . In all other cases  $R$  is equal to a constant  $c$  (which is in general not unique). If  $f'(c) \neq 0$ , eq. (11) implies that  $\Gamma$  is the Levi-Civita connection of  $g_{\mu\nu}$ , and (10) reduces to Einstein's vacuum equation with a cosmological constant. In general, one can rewrite the field equations in the form of Einstein gravity with nonstandard matter couplings. Because of this it is, for instance, straightforward to develop cosmological perturbation theory [32].

Koivisto [33] has applied this to study the resulting matter power spectrum, and showed that the comparison with observations leads to strong

constraints. The allowed parameter space for a model of the form  $f(R) = R - \alpha R^\beta$  ( $\alpha > 0$ ,  $\beta < 1$ ) is reduced to a tiny region around the  $\Lambda$ CDM cosmology.

The literature on this type of generalized gravity models is rapidly increasing.

### 6.3.3 Brane-world models

Certain brane-world models<sup>3</sup> lead to modifications of Friedmann cosmology at very large scales. An interesting example has been proposed by Dvali, Gabadadze and Porrati (DGP), for which the theory remains four-dimensional at ‘short’ distances, but crosses over to higher-dimensional behavior of gravity at some very large distance [34]. This model has the same number of parameters as the successful  $\Lambda$ CDM cosmology, but contains no Dark Energy. The resulting modified Friedmann equations can give rise to universes with accelerated expansion, due to an infrared modification of gravity.

In Ref. [36] the predictions of the model have been confronted with latest supernovae data [8], and the position of the acoustic peak in the SDSS correlation function for a luminous red galaxy sample [37]. The result is that a flat DGP brane model is ruled out at  $3\sigma$ . A similar analysis was more recently performed in [38], however using the SNe data [2], but including the CMB shift parameter that effectively determines the first acoustic peak (see Sect. 5.1). The authors arrive at the conclusion that the flat DGP models are within the  $1\sigma$  contours, but that the flat  $\Lambda$ CDM model provides a better fit to the data. They also point out some level of uncertainty in the use of the data, and conservatively conclude that the flat DGP models are within joint  $2\sigma$  contours.

This nicely illustrates that observational data are restricting theoretical speculations more and more.

The DGP models have, however, serious defects on a fundamental level. A detailed analysis of the excitations about the self-accelerating solution showed that there is a *ghost mode* (negative kinetic energy) [39]. Furthermore, it has very recently been pointed out [40] that due to superluminal fluctuations around non-trivial backgrounds, there is *no local causal evolution*. This infrared breakdown also happens for other apparently consistent low-energy effective theories.

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<sup>3</sup>For a review, see Ref. [35].



## 7 Has Dark Energy been discovered in the Lab?

It has been suggested by Beck and Mackey [41] that part of the zero-point energy of the radiation field that is gravitationally active can be determined from noise measurements of Josephson junctions. This caused some widespread attention. In a reaction we [42] showed that there is no basis for this claim, by following the reasoning in [41] for a much simpler model, for which it is very obvious that the authors misinterpreted their formulae. Quite generally, the absolute value of the zero-point energy of a quantum mechanical system has no physical meaning when gravitational coupling is ignored. All that is measurable are *changes* of the zero-point energy under variations of system parameters or of external couplings, like an applied voltage. For further information on the controversy, see [43] and [44].

\* \* \*

The previous discussion should have made it clear that it is extremely difficult to construct consistent modifications of GR that lead to an accelerated universe at late times. The Dark Energy problems will presumably stay with us for a long time. Understanding the nature of DE is widely considered as one of the main goals of cosmological research for the next decade and beyond.

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